Inverse Z-Transform

① $Z\{\delta[n]\} = 1$ $Z\{\delta[n-3]\} = z^{-3}$ $Z\{n\delta[n-1]\} = -z\frac{d}{dz}z^{-1} = -z\cdot(-z^{-2})$

$Z\{n\delta[n-1]\} = z^{-1}$

② $Z\{u[n]\} = \frac{z}{z-1}$

③ $Z\{0.3^n u[n]\} = \frac{z}{z-0.3}$

④ $Z\{n u[n]\} = \frac{z}{(z-1)^2}$

Exp/ Find $x[n]$ $X(z) = \frac{1-\frac{5}{6}z^{-1}}{(1-\frac{1}{4}z^{-1})(1-\frac{1}{3}z^{-1})} * z^2 * z^{-2}$

Sol. $X(z) = \frac{z^2 - \frac{5}{6}z}{(z-\frac{1}{4})(z-\frac{1}{3})}$, $\frac{X(z)}{z} = \frac{z-\frac{5}{6}}{(z-\frac{1}{4})(z-\frac{1}{3})}$

$\frac{X(z)}{z} = \frac{A}{z-\frac{1}{4}} + \frac{B}{z-\frac{1}{3}}$

$A = \lim_{z\to\frac{1}{4}} \left[\frac{z-\frac{5}{6}}{(z-\frac{1}{4})(z-\frac{1}{3})} * (z-\frac{1}{4})\right] = 7$

$B = \lim_{z\to\frac{1}{3}} \left[\frac{z-5/6}{(z-\frac{1}{4})(z-\frac{1}{3})} * (z-\frac{1}{3})\right] = -6$

$X(z) = \frac{7z}{z-\frac{1}{4}} - \frac{6z}{z-\frac{1}{3}}$

I-Z-T

$x[n] = 7\left(\frac{1}{4}\right)^n u[n] - 6\left(\frac{1}{3}\right)^n u[n]$

Exp/ Find $x[n]$

$$X(z)=\frac{z^4+z^2}{z^2-\frac{3}{4}z+\frac{1}{8}}$$

$$\frac{X(z)}{z}=\frac{z^3+z}{z^2-\frac{3}{4}z+\frac{1}{8}}$$

$$z^2-\frac{3}{4}z+\frac{1}{8}\;\overline{)\;z^3+z}\qquad z+\frac{3}{4}$$

$$-z^3+\frac{3}{4}z^2-\frac{1}{8}z$$

$$\frac{3}{4}z^2+\frac{7}{8}z$$

$$-\frac{3}{4}z^2+\frac{9}{16}z-\frac{3}{32}$$

$$\frac{23}{16}z-\frac{3}{32}$$

$$\frac{X(z)}{z}=z+\frac{3}{4}+\frac{\frac{23}{16}z-\frac{3}{32}}{z^2-\frac{3}{4}z+\frac{1}{8}}$$

$$\frac{X(z)}{z}=z+\frac{3}{4}+\frac{\frac{23}{16}z-\frac{3}{32}}{(z-\frac{1}{2})(z-\frac{1}{4})}$$

$$\frac{X(z)}{z}=z+\frac{3}{4}+\frac{A}{z-\frac{1}{2}}+\frac{B}{z-\frac{1}{4}}$$

$$A=\frac{5}{2}\qquad B=-\frac{17}{16}$$

$$X(z)=z^2+\frac{3}{4}z+\frac{\frac{5}{2}z}{z-\frac{1}{2}}+\frac{\frac{-17}{16}z}{z-\frac{1}{4}}$$

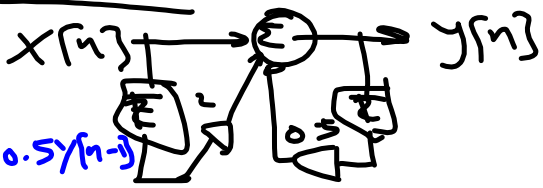
I.Z.T

$$x[n]=\delta[n+2]+\frac{3}{4}\delta[n+1]+\frac{5}{2}\cdot\left(\frac{1}{2}\right)^n u[n]+\frac{-17}{16}\left(\frac{1}{4}\right)^n u[n]$$

Z-Transform for discrete system

Exp/ For LTI system

① Find system stability



Sol.

$$y[n] = x[n] + 2x[n-1] + 0.5y[n-1]$$

Z.T

$$Y(z) = X(z) + 2z^{-1}X(z) + 0.5z^{-1}Y(z)$$

$$Y(z) - 0.5z^{-1}Y(z) = X(z) + 2z^{-1}X(z)$$

$$(1 - 0.5z^{-1})Y(z) = (1 + 2z^{-1})X(z)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{1 + 2z^{-1}}{1 - 0.5z^{-1}} = \frac{z + 2}{z - 0.5}$$

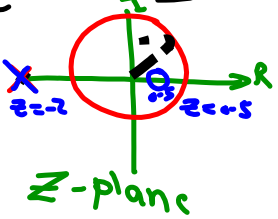
Zeros

$$z + 2 = 0 \Rightarrow z = -2$$

Poles

$$z - 0.5 = 0 \Rightarrow z = 0.5$$

System Stable



Exp2/ $y[n] = x[n] - 2x[n-2] + 0.5y[n-1] + 0.6y[n-2]$

Find System stability

sol/ $Y(z) = X(z) - 2z^{-2}X(z) + 0.5z^{-1}Y(z) - 0.6z^{-2}Y(z)$

$Y(z)(1 + 0.5z^{-1} - 0.6z^{-2}) = X(z)(1 - 2z^{-2})$

$H(z) = \frac{Y(z)}{X(z)} = \frac{1 - 2z^{-2}}{1 - 0.5z^{-1} + 0.6z^{-2}}$

$H(z) = \frac{z^2 - 2}{z^2 + 0.5z + 0.6}$

Zeros/ $z^2 - 2 = 0 \quad z = \pm\sqrt{2}$

Poly $z = \sqrt{2} \quad z = -\sqrt{2}$

$z^2 - 0.5z + 0.6 = 0$

$(z - 0.7)(z - 0.2) = 0$

$z = 0.7$

$z = 0.2$

System Stabl, $-\sqrt{2}$ $0.7, 0.2$ $\sqrt{2}$